Theoretical and Mathematical Physics, Vol. 118, No. 2, 1999

GRAVITATIONAL FIELD OF A FLAT SCALAR WAVE IN THE RELATIVISTIC THEORY OF GRAVITY

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We solve the relativistic theory of gravity equations in the case where the gravitational field source is a flat scalar wave and analyze the solution obtained.

1. Introduction

The relativistic theory of gravity (RTG) is based on viewing the gravitational field in the Faraday–Maxwell spirit as a physical field possessing energy and momentum [1, 2]. Thus, similarly to all other physical fields, the gravitational field is characterized by its energy–momentum tensor. Because the space–time geometry is pseudo-Euclidean (the Minkowski space) for all the physical fields, the density of a tensor is defined as $\widetilde{\Phi}^{\mu\nu} = \sqrt{-\gamma}\Phi^{\mu\nu}$, where $\gamma^{\mu\nu}$ is the metric tensor of the Minkowski space.

In complete analogy with the Maxwell electrodynamic equations in the absence of gravity, the gravitational field equations can be written in arbitrary coordinates as

$$\begin{aligned} &\gamma^{\alpha\beta}D_\alpha D_\beta\widetilde{\Phi}^{\mu\nu} + m^2\widetilde{\Phi}^{\mu\nu} = 16\pi t^{\mu\nu}, \\ &D_\mu\widetilde{\Phi}^{\mu\nu} = 0, \end{aligned} \tag{1}$$

where D_μ is the covariant derivative in Minkowski space and m is the graviton rest mass. For the gravitational field equations obtained from the minimum action principle to reduce to Eqs. (1), we must assume that the tensor density $\widetilde{\Phi}^{\mu\nu}$ always enters the Lagrangian together with the tensor density $\tilde{\gamma}^{\mu\nu}$ through a combined density $\tilde{g}^{\mu\nu}$, i.e.,

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \widetilde{\Phi}^{\mu\nu}, \qquad \tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}. \tag{2}$$

The motion of matter under the action of the gravitational field $\Phi^{\mu\nu}$ in the Minkowski space with the metric $\gamma_{\mu\nu}$ is identified in the RTG with the motion of matter in the effective Riemannian space whose metric $g^{\mu\nu}$ is determined by Eqs. (2). This interaction of the gravitational field with matter is called the *geometrization principle.* In the RTG, the equations of motion become

$$R_{\mu\nu} - \frac{m^2c^2}{2\hbar^2}(g_{\mu\nu} - \gamma_{\mu\nu}) = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right),$$

where $\hbar$ is the Planck constant and G is the gravitational constant. In this theory, therefore, the gravitational field is a Faraday–Maxwell field possessing energy, momentum, and spin 2 or 0 (these spin states are singled out by the second equation in system (1)). In accordance with condition (2), the Riemannian space arises as the effective space and has a field origin.

The nonvanishing rest mass of the graviton has a number of interesting consequences, one of which is the existence of gravitational repulsion forces from a singular sphere [3]. Exact solutions of the RTG equations with axial symmetry were recently found in [4]. The same symmetry is also possessed by the flat scalar wave. We now find the effective Riemannian space metric corresponding to the gravitational fields of the flat scalar wave.

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Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 118, No. 2, pp. 311–316, February, 1999. Original article submitted May 7, 1998.

 0040-5779/99/1182-0246$22.00

2. The flat-wave solution

As the matter generating the gravitational field, we consider a flat massless scalar wave $\phi(t, \mathbf{r})$. We write the matter part of the Lagrangian as

$$\mathcal{L}_{\mathrm{M}} = \sqrt{-g} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi.$$

With this choice of the gravitational field source, the RTG gravity equations become

$$\begin{aligned} &R_{\mu\nu} - \frac{m^2 c^2}{2\hbar^2}(g_{\mu\nu} - \gamma_{\mu\nu}) = -\frac{16\pi G}{c^4} \nabla_\mu \phi \nabla_\nu \phi, \\ &g^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0. \end{aligned} \tag{3}$$

The second equation in (3) is the scalar-field equation of motion.

We note that the scalar field is usually considered in the theory of gravity [5] as an auxiliary gravitational field. Here, we assume that the scalar field, along with other matter fields, is merely a source of the gravitational field.

We must solve the system of nonlinear equations given by Eqs. (3). If we are fortunate, we choose a method for seeking solutions whereby the system is reduced to a linear system.

We determine the metric of the effective Riemannian space from an analysis of the wave front of matter generating the gravitational field. It is known [6] that the wave front equation for Eq. (3) is $g^{\mu\nu}\partial_\mu S\, \partial_\nu S = 0$. This is the equation of motion for a massless particle in the Hamilton–Jacobi form, with S being the eikonal action of the field quantum.

We require that the front of our scalar wave be the same in the Minkowski space and in the effective Riemannian space; that is, we require that the equations $g^{\mu\nu}\partial_\mu S \partial_\nu S = 0$ and $\gamma^{\mu\nu}\partial_\mu S \partial_\nu S = 0$ be satisfied simultaneously. This requirement imposes severe constraints not only on the form of the sought-for metric $g^{\mu\nu}$ but also (and primarily) on the form of the scalar field $\phi(x)$. We now represent the metric $g^{\mu\nu}$ of the effective Riemannian space as $g^{\mu\nu} = \gamma^{\mu\nu} + \xi^{\mu\nu}$. Then our constraint becomes the solvability requirement for the system of equations

$$\begin{aligned} &\gamma^{\mu\nu}\partial_\mu S \partial_\nu S = 0, \\ &\xi^{\mu\nu}\partial_\mu S \partial_\nu S = 0. \end{aligned} \tag{4}$$

From this moment on, we work in Cartesian coordinates. We assume that the flat wave propagates along the Z axis. The action S is then the function $S = S(z, ct)$, and the first equation in (4) implies that $\partial_{ct} S = \pm \partial_z S$ or equivalently $S = S(ct \pm z)$. The sign $\pm$ corresponds to the wave propagation direction, and we choose $S = S(ct - z)$ for definiteness. Then, inserting this into the second equation in (4), we obtain

$$\frac{dS(ct - z)}{d(ct - z)}\left(\xi^{00} - 2\xi^{03} + \xi^{33}\right) = 0. \tag{4a}$$

We set $\xi^{00} = \xi^{03} = \xi^{33} = -F(x^\lambda)$ and $\xi^{\alpha\beta} = 0$ for all the other components. Then the left-hand side of Eq. (4a) vanishes automatically.

We note that the function F depends on x^λ as $F(x^\lambda) = F(ct - z)$. This dependence is determined by the form of the metric tensor $g_{\mu\nu} = g_{\mu\nu}(ct - z)$, which in turn is a direct consequence of the form of the function $S = S(ct - z)$. Therefore, the tensor $g_{\mu\nu}(x^\lambda)$ is

$$g_{\mu\nu} = \begin{pmatrix} 1 + F(ct - z) & 0 & 0 & -F(ct - z) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -F(ct - z) & 0 & 0 & -1 + F(ct - z) \end{pmatrix}, \tag{5}$$

and the effective metric tensor (in fact, the gravitational wave) is a certain perturbation $F(ct-z)$ of the pseudo-Euclidean metric propagating along the Z axis with the speed c.

As noted above, this form of the metric imposes severe constraints on the form of the scalar field. These constraints, obviously, follow from equation of motion (3) for the the scalar field. Indeed, this is the only equation that relates the quantities ϕ and $g^{\mu\nu}$ and does not contain other quantities. The sought-for condition is that this equation be satisfied for all ϕ.

Expressing the second equation in (3) as

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}g^{\mu\nu}\partial_k\phi)}{\partial x^i}=0,$$

we rewrite it in components as

$$\phi_{zz}(F-1)+\phi_{ct\,ct}(F+1)+2F\phi_{z\,ct}=0.$$

Inserting ϕ as $\phi=\phi(\zeta(ct,z))$, we obtain the equation for ζ,

$$\phi_{\zeta\zeta}\left(\zeta_z^2(F-1)+\zeta_{ct}^2(F+1)+\zeta_z\zeta_{ct}F\right)+\phi_\zeta\left(\zeta_{zz}(F-1)+\zeta_{ct\,ct}(F+1)+\zeta_{z\,ct}F\right)=0.$$

Now assuming ζ to be a linear function, we set $\alpha\zeta_z=\zeta_{ct}$, and therefore $F(1+\alpha^2+\alpha)+\alpha^2-1=0$, whence $\alpha=-1$, which means that the dependence $\zeta(ct,z)=\zeta(ct-z)$ makes the second equation in (3) an identity. Thus, we have shown that $\phi=h(ct-z)$, where $h(ct-z)$ is an arbitrary function describing a particular scalar-field wave packet propagating along the Z axis with the speed c.

Such a field generates a perturbation of the flat metric that also moves along the Z axis with the speed c. As a result, Eqs. (3) allow us to obtain the equation

$$-2\left(\phi'_{(ct-z)}\right)^2\frac{16\pi G}{c^4}-\frac{m^2c^2}{2\hbar^2}F=0.$$

This is solved by

$$F(ct-z)=-\frac{64\pi G\hbar^2}{m^2c^8}\left(\frac{\partial\phi(ct-z)}{\partial t}\right)^2. \tag{6}$$

We have thus found the exact form of the effective Riemannian space metric $g^{\mu\nu}$ and hence the exact form of the gravitational field tensor density $\widetilde{\Phi}^{\mu\nu}$.

3. The analysis of the solution

We first check whether the solution that we obtained in Sec. 2 satisfies the causality principle [1]: if $d\sigma^2=\gamma_{\mu\nu}dx^\mu dx^\nu=0$, then $ds^2=g_{\mu\nu}dx^\mu dx^\nu\leq 0$. Inserting the metric obtained above into these conditions demonstrates that they are completely satisfied by virtue of $F(ct-z)\leq 0$. Therefore, the solution obtained satisfies this causality principle and is thus "physical."

The curvature tensor allows us to elucidate the gravitational wave geometry. However, the tensor $R_{\mu\nu\sigma\lambda}$ constructed from the metric obtained above vanishes, which means that the wave does not give rise to a curved geometry. Therefore, the action of the gravitational wave obtained above reduces to the following effect: the passing gravitational wave can be considered to accelerate the coordinate system obtained in metric (5), making it noninertial in the pseudo-Euclidean space-time. When the wave has gone, the coordinate system returns to the initial state, becoming inertial again.

We now find the explicit form of the coordinate transformations taking the accelerated reference frame into the noninertial one. Because the acceleration occurs along the propagation direction of the scalar wave (the Z axis in our case), we consider the transformations of only two coordinates

$$T=T(z,t),\qquad Z=Z(z,t),$$

where T and Z denote Minkowski space coordinates. We seek transformations in the form (setting $c \equiv 1$)

$$dT = \big(1 + a_1 F(t-z)\big)dt - a_1 F(t-z)dz,$$
$$dZ = a_2 F(t-z)dt + \big(1 - a_2 F(t-z)\big)dz,$$

where a_1 and a_2 are constants. It is clear that the conditions

$$\frac{\partial^2 T}{\partial z\,\partial t} = \frac{\partial^2 T}{\partial t\,\partial z}, \qquad \frac{\partial^2 Z}{\partial z\,\partial t} = \frac{\partial^2 Z}{\partial t\,\partial z}$$

are then satisfied and we have $dX^\alpha = dx^\alpha$ in the absence of the gravitational field.

Writing out the Minkowski space interval

$$d\sigma^2 = dT^2 - dX^2 - dY^2 - dZ^2$$

in terms of the Riemannian coordinates and comparing it with the effective Riemannian space interval

$$ds^2 = dt^2\big(1 + F(t-z)\big) - dz^2\big(1 - F(t-z)\big) - 2dt\,dz\,F(t-z) - dx^2 - dy^2,$$

we see that each constant a_1 and a_2 is equal to $1/2$. The desired transformations therefore become

$$\begin{aligned} dT &= \left(1 + \frac{1}{2}F(t-z)\right)dt - \frac{1}{2}F(t-z)dz, \\ dX &= dx, \\ dY &= dy, \\ dZ &= \frac{1}{2}F(t-z)dt + \left(1 - \frac{1}{2}F(t-z)\right)dz. \end{aligned} \tag{7}$$

We now consider the equation of motion for a fixed point (for example, $x = y = z = 0$) in the inertial reference frame. According to (7), we have

$$\begin{aligned} T &= \int dt\left\{1 + \frac{1}{2}F(t)\right\}, \\ Z &= \frac{1}{2}\int dt\,F(t), \\ X &= 0, \\ Y &= 0, \end{aligned} \tag{8}$$

where time t can be considered a parameter entering the parametric form of the equations of motion for the functions $T(t)$ and $Z(t)$. We see from (8) that

$$\frac{d^2 Z}{dT^2} = \frac{4\dot{F}(t)}{(2 + F(t))^3} \neq 0$$

if $F(t) \neq -2$ holds.[2] Therefore, in view of Eqs. (5) and (6), the reference frame acceleration with respect to the inertial frame is nonzero if $\partial^2\phi/\partial t^2 \neq 0$ for the time-dependent scalar field.

The main result of this work can therefore be formulated as follows. *An arbitrary scalar-field wave packet propagating in a given direction induces a gravitational field resulting in a variable acceleration of the reference frame with respect to the inertial reference frame in the Minkowski space.*

[2]We note that this constraint is always satisfied because it follows from the form of metric (5) that the value of F cannot be less than -1; this indicates that the scalar field ϕ is small.

Acknowledgments. This work was supported by the Russian Foundation for Basic Research (Grant No. 96-15-96674).

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